12.3 Permutations and Combinations

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There are 6 people who want to use an elevator. There is only room for 4 people. How many ways can 6 people try to fill this elevator (one at a time)?

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<u>6 x 5 x 4 x 3</u>

= 360

Permutations

DEFINITION A **permutation** is an ordering of distinct objects in a straight line. If we select *r* different objects from a set of *n* objects and arrange them in a straight line, this is called a *permutation of n objects taken r at a time.* The number of permutations of *n* objects taken *r* at a time is denoted by P(n, r).



There are 6 people who want to use an elevator. There is only room for 4 people. How many ways can 6 people try to fill this elevator (one at a time)?

P(6, 4) =

<u>6 x 5 x 4 x 3</u>

= 360

Permutations

• Example: How many permutations are there of the letters a, b, c, d, e, f, and g if we take the letters three at a time? Write the answer using P(n, r) notation.

Permutations

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P(n,r) describes a slot diagram.

n = number in first slot r = number of slots



$$P(7, 3) = 210$$
number of -1 taken 3 at a time permutations of 7 objects

How many ways are there to arrange 5 books on a bookshelf?

How many ways are there to arrange 5 books on a bookshelf?

P(5,5) =

<u>5 x 4 x 3 x 2 x 1</u>

= 120

Shortcut/Defintion

DEFINITION If *n* is a counting number, the symbol *n*!, called *n* factorial, stands for the product $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \cdots \cdot 2 \cdot 1$. We define 0! = 1.

Example: 5! = 5x4x3x2x1



Compute (5 - 2)!

Example:

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$$(5-2)! = 3! = 3x2x1 = 6$$

Factorial Notation

• Example: Compute $\frac{8!}{5!3!}$.

Factorial Notation

- Example: Compute $\frac{8!}{5!3!}$.
- Solution:

$$\frac{8!}{5!3!} = \frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 8 \cdot 7 = 56.$$

Cancel 5!. Cancel 3!, which equals 6.

Factorial Notation

FORMULA FOR COMPUTING P(n, r)

$$P(n, r) = \frac{n!}{(n-r)!}$$

There are 6 people who want to use an elevator. There is only room for 4 people. How many ways can 6 people try to fill this elevator (one at a time)?

P(6, 4) =

= 360

When we care about the order of objects, like books on a bookshelf, we have a <u>permutation</u>.

When we do not care about the order of objects, like 2 people wining a raffle, we have a <u>combination</u>.

FORMULA FOR COMPUTING C(n, r) If we choose *r* objects from a set of *n* objects, we say that we are forming a **combination** of *n* objects taken *r* at a time. The notation C(n, r) denotes the number of such combinations.[†] Also,

$$C(n, r) = \frac{P(n, r)}{r!} = \frac{n!}{r! \cdot (n-r)!}.$$

Example: A person would like to run 4 errands, but only has time for 2. How many pairs of errands could be tried?

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Order does not matter = combination.

$$C(4,2) = \frac{4!}{(4-2)! \ 2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1} = 6$$

- Example: How many three-element sets can be chosen from a set of five objects?
- Solution: Order is not important, so it is clear that this is a combination problem.

$$C(5,3) = \frac{5!}{3! \cdot (5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{20}{2} = 10$$

• Example: How many four-person committees can be formed from a set of 10 people?

- Example: How many four-person committees can be formed from a set of 10 people?
- Solution: Order is not important, so it is clear that this is a combination problem.

$$C(10,4) = \frac{10!}{4! \cdot (10-4)!} = \frac{10 \cdot \cancel{9} \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 210$$

Example: At a vation spot there are 7 sites to visit, but you only have time for 5. How many different combinations do you have to choose from? Example: At a vation spot there are 7 sites to visit, but you only have time for 5. How many different combinations do you have to choose from?

Order does not matter = combination.

C(7,5) = 21

Combining counting methods.

Sometimes you will have more than one counting idea to find the total number of possibilities.

Example: 2 men and 2 women from a firm will attend a conference. The firm has 10 men and 9 women to choose from. How many group possibilities are there? Example: 2 men and 2 women from a firm will attend a conference. The firm has 10 men and 9 women to choose from. How many group possibilities are there?

1st task : choose 2 men from 10

2nd task : choose 2 women from 10

Use a slot diagram $x = 1^{st} 2^{nd}$

Combining Counting Methods

Stage 1: Select the two women from the nine available. $C(0, 2) = \frac{9!}{26} = \frac{26}{100}$

 $C(9, 2) = \frac{9!}{2!7!} = 36$ ways

Stage 2: Select the two men from the ten available. $C(10, 2) = \frac{10!}{2!8!} = 45$ ways

Thus, choosing the women and then choosing the men can be done in $36 \cdot 45 = 1,620$ ways.

Example: How many different outcomes are there for rolling a die and then drawing 2 cards from a deck of cards? Example: How many different outcomes are there for rolling a die and then drawing 2 cards from a deck of cards?

1st task : roll a die

2nd task : draw 2 cards from 52 (order does not matter)

Use a slot diagram $x_{1^{st}} x_{2^{nd}}$

Example: How many different outcomes are there for rolling a die and then drawing 2 cards from a deck of cards?

1st task : roll a die = 6 ways

 2^{nd} task : draw 2 cards from 52 = C(52,2) (order does not matter)

Use a slot diagram
$$\frac{6}{1^{st}} \times \frac{1326}{2^{nd}} = 7956$$